

Application of Econometric Techniques to the Determination of Property Prices – Part 1

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Introduction

□ Econometric techniques can be used to analyse the:

- relationship between different property investments.**
- relationship between property assets and other assets.**
- relationship between property investment returns and inflation.**
- relationship between the property market and the general economy**

Techniques for investigating the relationship between variables

- **Covariance Analysis**
- **Correlation Analysis**
- **Regression Analysis**
- **Cointegration Analysis**
- **Causality Analysis**
- **Vector Autoregression**

COVARIANCE

- **Covariance is a measure of the joint variability of two random variables. In other words, it is a measure of the direction of the relationship between two variables.**
- **A positive covariance implies that greater values of one variable correspond with greater values of the other variable and vice-versa.**
- **Covariance can provide an idea of the direction of the relationship but not the strength of the relationship.**

SAMPLE COVARIANCE

$$\text{Cov}_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

Where:

- Cov_{xy} = Covariance between variable x and y
- $\bar{x}$ = mean of variable x
- $\bar{y}$ = mean of variable y
- n = Number of observations (Sample size)

POPULATION COVARIANCE

$$\text{Cov}_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{N}$$

Where:

- Cov_{xy} = Covariance between variable x and y
- $\bar{x}$ = mean of variable x
- $\bar{y}$ = mean of variable y
- N = Number of observations (Population size)

Example 1

- **The table below presents the stock prices for two listed property funds: Addis Realty and Pineland Development.**

Addis Realty (x)	17	13	12	15	16
Pineland Development (y)	94	73	59	80	93

- **Calculate the covariance between the stock prices of the two funds.**

Example 2 - Answer

	x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$	$(y - \bar{y})^2$
	17	94	2.4	14.2	34.08	5.76	201.64
	13	73	-1.6	-6.8	10.88	2.56	46.24
	12	59	-2.6	-20.8	54.08	6.76	432.64
	15	80	0.4	0.2	0.08	0.16	0.04
	16	93	1.4	13.2	18.48	1.96	174.24
Mean	14.6	79.8		$\Sigma =$	117.6	17.2	854.8

Example 2 (Answer)

Covariance	
$\sum(x-\bar{x})(y-\bar{y})$	117.6
n	5
n-1	4
$COV_{x,y} = \sum(x-\bar{x})(y-\bar{y})/n-1$	29.4
Covariance using excel, Enter “=covariance.s(x,y)”	29.4

Example 2

- **The table below presents the returns of two listed property companies, Newlands REIT and Rosebank Property Inc.**

Newlands REIT (x)	5	7	8	4	9	3
Rosebank Property Inc. (y)	2	4	5	5	6	5

- **Calculate the covariance between the returns of the two companies.**

Example 2 - Answer

	x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$	$(y - \bar{y})^2$
	5	2	-1	-2.5	2.5	1	6.25
	7	4	1	-0.5	-0.5	1	0.25
	8	5	2	0.5	1	4	0.25
	4	5	-2	0.5	-1	4	0.25
	9	6	3	1.5	4.5	9	2.25
	3	5	-3	0.5	-1.5	9	0.25
Mean	6	4.5			5	28	9.5

Example 2 (Answer)

Covariance	
$\Sigma(x-\bar{x})(y-\bar{y})$	5
n	6
n-1	5
$COV_{x,y} = \Sigma(x-\bar{x})(y-\bar{y})/n-1$	1
$COV_{x,y}$ using excel, Enter “=covariance.s(x,y)”	1

CORRELATION ANALYSIS

- **Correlation analysis is used to determine the degree or strength of association (relationship) between variables.**
- **Correlation also shows the direction of association i.e. whether there is a positive or negative relationship between the variables.**
- **Note however, that correlation does not imply causation.**
- **Types: Pearson Correlation, Spearman Rank Correlation**

Pearson Correlation – Formula 1

$$\rho = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \times \sqrt{\sum (y_i - \bar{y})^2}}$$

- Where:
- $\bar{x}$ = mean of x variable
- $\bar{y}$ = mean of y variable

Pearson Correlation – Formula 2

- **The correlation coefficient can also be estimated using the following relation:**

$$\rho = \frac{Cov_{xy}}{\sigma_x \times \sigma_y}$$

- **Where:**
 - Cov_{xy} = covariance between x and y
 - σ_x = standard deviation of x
 - σ_y = standard deviation of y

Pearson Correlation – Formula 2 cont.

- **Note that, the Pearson Correlation can be calculated using either the sample or population standard deviation and covariance values.**
- **It is important however to be consistent with the choice.**
- **For example, when you use the sample covariance, you should also use the sample standard deviation.**
- **If done correctly, both approaches should produce the same correlation coefficient.**

Pearson Correlation Coefficient

- **The value of the correlation coefficient ranges from -1 and 1.**
- **A correlation coefficient of 1 means that with every increase in one variable, there is a positive proportional change in the other variable.**
- **A correlation coefficient of -1 means that for every positive increase in one variable, there is a negative proportional change in the other variable.**
- **A correlation of zero (0) implies that the two variables are not related.**

Example 1

- **The table below presents the stock prices for two listed property companies Addis Realty and Pineland Development Limited.**

Addis Realty (X)	17	13	12	15	16
Pineland Development (Y)	94	73	59	80	93

- **Estimate the correlation between the stock prices of the two companies.**

Example 1 - Answer

	x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$	$(y - \bar{y})^2$
	17	94	2.4	14.2	34.08	5.76	201.64
	13	73	-1.6	-6.8	10.88	2.56	46.24
	12	59	-2.6	-20.8	54.08	6.76	432.64
	15	80	0.4	0.2	0.08	0.16	0.04
	16	93	1.4	13.2	18.48	1.96	174.24
Mean	14.6	79.8			117.6	17.2	854.8

Example 1 - Answer

Correlation using formula 1

$\sum(x-\bar{x})(y-\bar{y}) =$	117.60
$\sum(x-\bar{x})^2 =$	17.20
$\sqrt{\sum(x-\bar{x})^2} =$	4.15
$\sum(y-\bar{y})^2 =$	854.8
$\sqrt{\sum(y-\bar{y})^2} =$	29.24
Correlation (ρ) = $\sum(x-\bar{x})(y-\bar{y}) / \{[\sqrt{\sum(x-\bar{x})^2}] * [\sqrt{\sum(y-\bar{y})^2}]\}$	0.9699
Correlation using excel: Enter “=correl(x,y)” =	0.9699

Example 1 - Answer

Correlation using formula 2	
Standard deviation of x:	
$\sum(x-\bar{x})^2 =$	17.2
$\sum(x-\bar{x})^2/(n-1) =$	4.3
$\sigma_x = \sqrt{[\sum(x-\bar{x})^2/(n-1)]} =$	2.0736
Using excel: Enter “=stdev.s($x_i:x_n$)” =	2.0736
Standard deviation of y:	
$\sum(y-\bar{y})^2 =$	854.8
$\sum(y-\bar{y})^2/n-1 =$	213.7
$\sigma_y = \sqrt{[\sum(y-\bar{y})^2/n-1]} =$	14.6185
Using excel: Enter “=stdev.s($y_i:y_n$)” =	14.6185
$\rho = \text{Covariance}(x,y)/\sigma_x\sigma_y =$	0.9699

Example 2

- **The table below presents the returns of two listed property companies Newlands REIT and Rosebank Property Inc.**

Newlands REIT (X)	5	7	8	4	9	3
Rosebank Property Inc. (Y)	2	4	5	5	6	5

- **Estimate the correlation between the returns of the two companies.**

Example 2 - Answer

	x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$	$(y - \bar{y})^2$
	5	2	-1	-2.5	2.5	1	6.25
	7	4	1	-0.5	-0.5	1	0.25
	8	5	2	0.5	1	4	0.25
	4	5	-2	0.5	-1	4	0.25
	9	6	3	1.5	4.5	9	2.25
	3	5	-3	0.5	-1.5	9	0.25
Mean =	6	4.5		$\Sigma =$	5	28	9.5

Example 2 - Answer

Correlation using formula 1

$\Sigma(x-\bar{x})(y-\bar{y}) =$	5.0000
$\Sigma(x-\bar{x})^2 =$	28.0000
$\sqrt{[\Sigma(x-\bar{x})^2]} =$	5.2915
$\Sigma(y-\bar{y})^2 =$	9.5000
$\sqrt{[\Sigma(y-\bar{y})^2]} =$	3.0822
Correlation (ρ) = $\Sigma(x-\bar{x})(y-\bar{y}) / \{[\sqrt{[\Sigma(x-\bar{x})^2]} * \sqrt{[\Sigma(y-\bar{y})^2]}]\} =$	0.3066
Correlation using excel: Enter “=correl(x,y)”	0.3066

Example 2 - Answer

Correlation using formula 2	
Standard deviation of x:	
$\sum(x-\bar{x})^2 =$	28.0000
$\sum(x-\bar{x})^2/(n-1) =$	5.6000
$\sigma_x = \sqrt{[\sum(x-\bar{x})^2/(n-1)]} =$	2.3664
Using excel: Enter “=stdev.s($x_i:x_n$)” =	2.3664
Standard deviation of y:	
$\sum(y-\bar{y})^2 =$	9.5000
$\sum(y-\bar{y})^2/n-1 =$	1.9000
$\sigma_y = \sqrt{[\sum(y-\bar{y})^2/n-1]} =$	1.3784
Using excel: Enter “=stdev.s($y_i:y_n$)” =	1.3784
Correlation (ρ) = $COV_{x,y} / (\sigma_x * \sigma_y)$	0.3066
Correlation using excel: Enter “=correl(x,y)” =	0.3066

Example 3

- **The table below presents stock prices for two listed property companies Addis Realty and Pineland Development Limited.**

Addis Realty (X)	17	13	12	15	16	14	16	16	18	19
Pineland Development (Y)	94	73	59	80	93	85	66	79	77	91

- **Estimate the correlation coefficient between the stock prices of the two companies.**



Example 3 (Answer)

	x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$
	17	94	1.4	14.3	20.02	1.96	204.49
	13	73	-2.6	-6.7	17.42	6.76	44.89
	12	59	-3.6	-20.7	74.52	12.96	428.49
	15	80	-0.6	0.3	-0.18	0.36	0.09
	16	93	0.4	13.3	5.32	0.16	176.89
	14	85	-1.6	5.3	-8.48	2.56	28.09
	16	66	0.4	-13.7	-5.48	0.16	187.69
	16	79	0.4	-0.7	-0.28	0.16	0.49
	18	77	2.4	-2.7	-6.48	5.76	7.29
	19	91	3.4	11.3	38.42	11.56	127.69
Average =	15.6	79.7			134.8	42.4	1,206.1

Example 3 (Answer)

Correlation using formula 1

$$\sum (x - \bar{x})(y - \bar{y}) = 134.8$$

$$\sum (x - \bar{x})^2 (y - \bar{y})^2 = 51138.64$$

$$\sqrt{[\sum (x - \bar{x})^2 (y - \bar{y})^2]} = 226.14$$

$$\text{Correlation } (\rho) = \sum (x - \bar{x})(y - \bar{y}) / \{[\sqrt{[\sum (x - \bar{x})^2]} * \sqrt{[\sum (y - \bar{y})^2]}\} = 0.5961$$

Covariance/Correlation Analysis in EViews

- **STEP 1**
 - **Load data into EViews**
- **STEP 2**
 - **Select variables and open as group**
- **STEP 3**
 - **Go to VIEW then select COVARIANCE ANALYSIS**
- **STEP 4**
 - **In LAYOUT, select MULTIPLE TABLES**
- **Click OK**

Example 3 - Answer

Correlation using formula 2	
Standard deviation of x:	
$\sum(x-\bar{x})^2 =$	42.4000
$\sum(x-\bar{x})^2/(n-1) =$	4.7111
$\sigma_x = \sqrt{[\sum(x-\bar{x})^2/(n-1)]} =$	2.1705
Using excel: Enter “=stdev.s(x _i :x _n)” =	2.1705
Standard deviation of y:	
$\sum(y-\bar{y})^2 =$	1206.1000
$\sum(y-\bar{y})^2/n-1 =$	134.0111
$\sigma_y = \sqrt{[\sum(y-\bar{y})^2/n-1]} =$	11.5763
Using excel: Enter “=stdev.s(y _i :y _n)” =	11.5763
Correlation (ρ) = $COV_{x,y} / (\sigma_x * \sigma_y)$	0.5961
Correlation using excel: Enter “=correl(x,y)” =	0.5961

Covariance/Correlation Analysis in Excel

- **Go to Data > Data Analysis**
- **Select: Correlation or Covariance**
- **Select input range (both X and Y)**
- **Select Labels if selection includes the label**
- **Select output range**
- **Click OK**

Example

- **Using the data linked [here](#), construct a covariance and correlation matrix for the following variables using MS Excel and EViews:**
 - a) Addis and Pinelands (5 year)**
 - b) Addis and Pinelands (10 year)**
 - c) Newland and Rosebank**
 - d) IPD_SA, SAPY, SA_CPI, SA_HPI.**

SIMPLE LINEAR REGRESSION

- **Regression is a technique used to describe the relationship between two or more variables in mathematical terms.**
- **Regression analysis examines the influence of one or more independent variables on a dependent variable.**
- **Regression can be used for estimation (forecasting).**

Simple Linear Regression - Formula

- **Linear regression function:**

$$y = \alpha + \beta x$$

- **Slope (b) of regression line:**

$$\beta = \rho \frac{\sigma_y}{\sigma_x} \quad \text{or} \quad \beta = \frac{COV_{xy}}{\sigma_x^2}$$

- **Y-intercept**

$$\alpha = \bar{y} - \beta \bar{x}$$

Simple Linear Regression – Using Excel

- **Go to Data > Data Analysis**
- **Select: Regression**
- **Input Y Range**
- **Input X Range**
- **Select Labels if selection includes the label**
- **Select output range**
- **Click OK**

Example 1

- **The table below presents the returns of two listed property companies Newlands REIT and Rosebank Property Inc.**

Newlands REIT (X)	5	7	8	4	9	3
Rosebank Property Inc. (Y)	2	4	5	5	6	5

- **Estimate a simple linear regression using the data above.**

Example 1 - Answer

	x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$	$(y - \bar{y})^2$
	5	2	-1	-2.5	2.5	1	6.25
	7	4	1	-0.5	-0.5	1	0.25
	8	5	2	0.5	1	4	0.25
	4	5	-2	0.5	-1	4	0.25
	9	6	3	1.5	4.5	9	2.25
	3	5	-3	0.5	-1.5	9	0.25
Mean	6	4.5			5	28	9.5

Example 1 - Answer

Correlation using formula 1	
$\sum(x-\bar{x})(y-\bar{y})$	5.0000
$\sum(x-\bar{x})^2$	28.0000
$\sqrt{[\sum(x-\bar{x})^2]}$	5.2915
$\sum(y-\bar{y})^2$	9.5000
$\sqrt{[\sum(y-\bar{y})^2]}$	3.0822
Correlation (ρ) = $COV_{x,y} / (\sigma_x * \sigma_y)$	0.3066

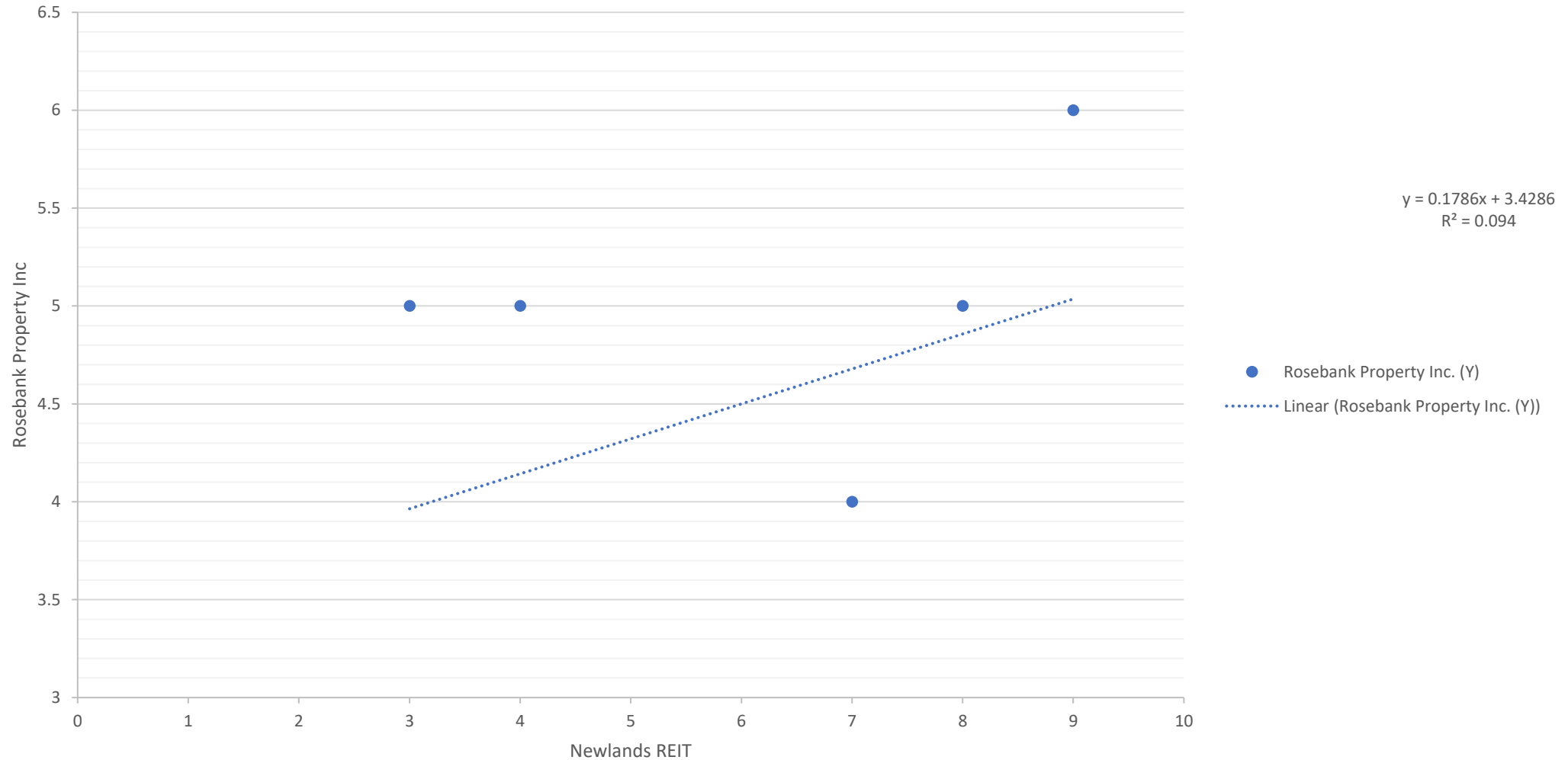
Example 1 - Answer

Slope (β):	
Correlation (ρ)	0.3066
Standard deviation of x (σ_x)	2.3664
Standard deviation of y (σ_y)	1.3784
Slope (β) = $\rho^*(\sigma_y/\sigma_x)$	0.1786
Intercept (α):	
$\bar{x}$	6.0000
$\bar{y}$	4.5000
Slope (β)	0.1786
Intercept (α) = $\bar{y} - \beta\bar{x}$	3.4286
Regression Equation: $y = 3.4286 + 0.1786x$	

Example 1 – Answer (Using Excel)

<i>Regression Statistics</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	0.8929	0.8929	0.4149	0.5546			
Residual	4	8.6071	2.1518					
Total	5	9.5						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	3.4286	1.7678	1.9394	0.1245	-1.4797	8.3368	-1.4797	8.3368
Newlands REIT (X)	0.1786	0.2772	0.6442	0.5546	-0.5911	0.9483	-0.5911	0.9483

Example 1 – Answer (Using Excel)

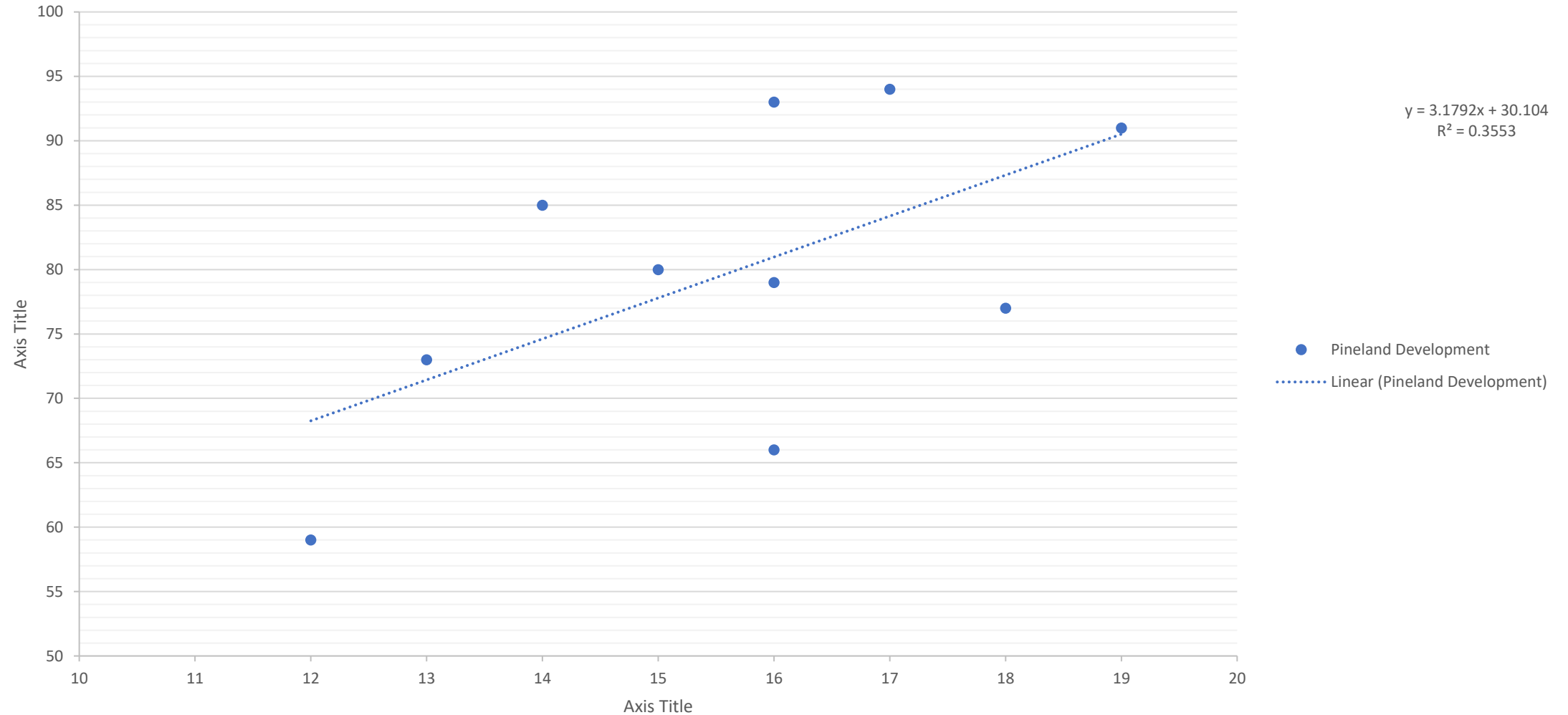


Example 1 – Answer (Using Excel)

Regression Statistics

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	428.5623	428.5623	4.4094	0.0690			
Residual	8	777.5377	97.1922					
Total	9	1206.1						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	30.1038	23.8236	1.2636	0.2419	-24.8336	85.0412	-24.8336	85.0412
Addis Realty	3.1792	1.5140	2.0999	0.0690	-0.3121	6.6706	-0.3121	6.6706

Example 1 – Answer (Using Excel)



Example 2

- **You have data on the building areas and prices of a number of properties advertised on an online property platform. The data is linked below:**

- **Data**

- a) **Using this data, estimate a simple linear regression to determine the relationship between building area and house prices.**
- b) **Based on the regression analysis, estimate how much properties with the following dimensions will sell for: (i) 120 (i) 900 (ii) 1400**

Example 3

- **Using the data linked [here](#):**
 - a) Estimate a simple linear regression model to demonstrate the impact of inflation on SA direct property returns.**
 - b) Estimate a simple linear regression model to demonstrate the impact of inflation on SA listed property returns.**
 - c) Estimate a simple linear regression model to demonstrate the impact of inflation on SA House prices.**

SIMPLE LINEAR REGRESSION IN EViews

- **STEP 1**

- **Load data into EViews**

- **STEP 2**

- **Select Y variable first, then select X variable and open as equation**

- **STEP 3**

- **In METHOD dropdown, select LEAST SQUARES.**

- **STEP 4**

- **Click OK**