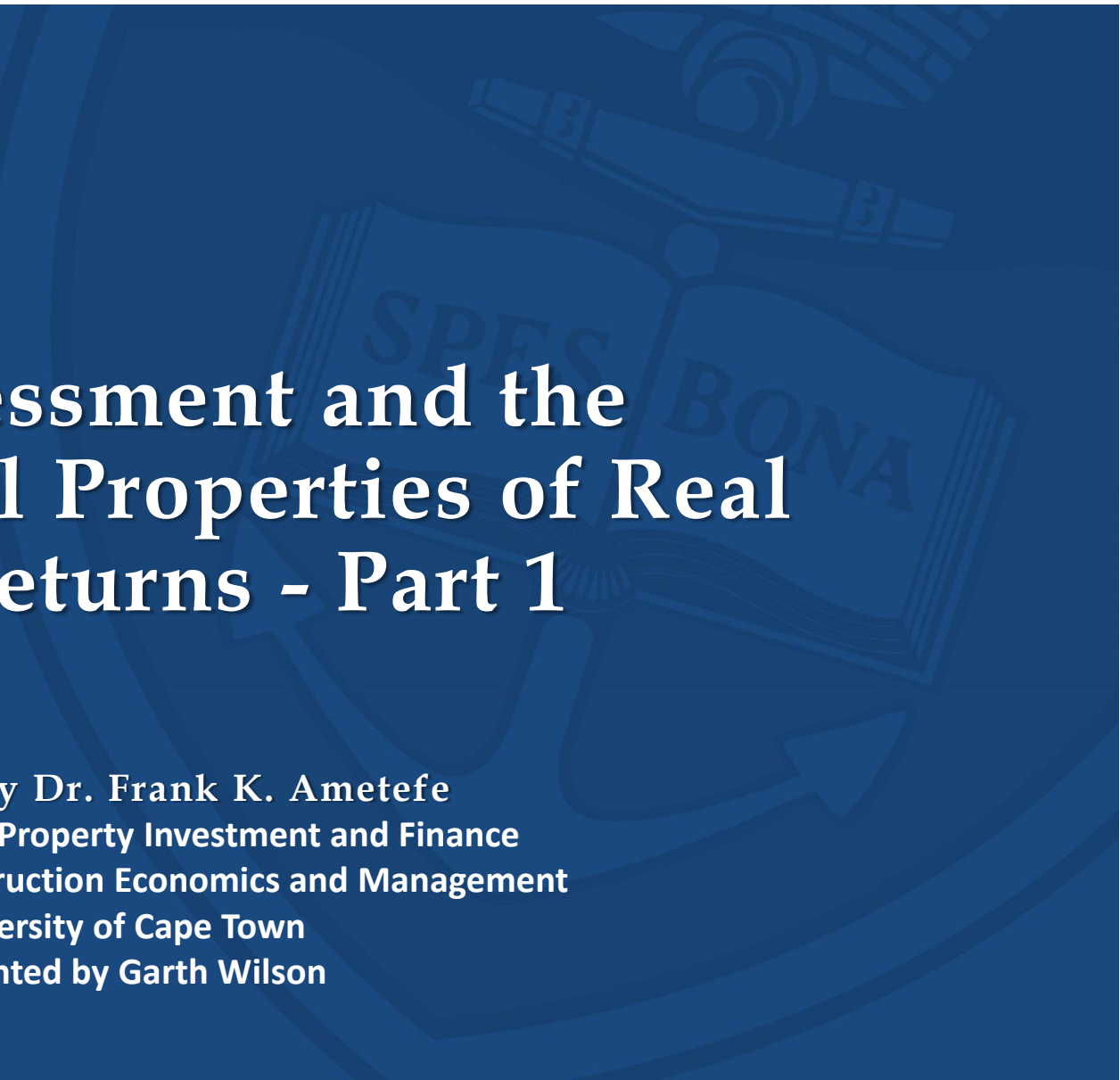


The background of the slide features a large, faint watermark of the University of Cape Town crest. The crest is circular and contains a shield with a book and a quill. The Latin motto 'SPES BONA' is visible on the book. The entire slide has a dark blue background.

Risk Assessment and the Distributional Properties of Real Estate Returns - Part 1

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What is risk?

Risk can impact all components of return through:

Fluctuations in income returns

Fluctuations in price changes of the investment

Fluctuations in reinvestment rates of return

Sources of Risk

Systematic Risk Factors

Affect many investment returns simultaneously; their impact is pervasive.

Examples: changes in interest rates, and the state of the macro-economy.

Sources

Currency risk

Inflation

Interest rate

Political risk

Sources of risk

Asset-specific Risk Factors

Affect only one or a small number of investment returns; come from the characteristics of the specific investment.

Examples: poor management, labor strikes, deterioration of product/service quality.

Sources

Business risk

Financial risk

Liquidity risk

Default risk

Variance for a historical rate of return

Sample Variance:

$$\sigma^2 = \frac{\sum (r - \bar{r})^2}{n-1}$$

Population Variance:

$$\sigma^2 = \frac{\sum (r - \bar{r})^2}{N}$$

Standard Deviation of Historic Returns

The standard deviation gives the variability of a distribution about the mean in the units being measured rather than in squared units as does the variance.

Thus, The standard deviation is the square root of the variance.

Standard Deviation of Historic Returns

Sample Standard Deviation:

$$\sigma = \sqrt{\frac{\sum (r - \bar{r})^2}{n-1}}$$

Population Standard Deviation:

$$\sigma = \sqrt{\frac{\sum (r - \bar{r})^2}{N}}$$

Example 1

Year	Return
1	10%
2	30%
3	-20%
4	0%
5	20%

Calculate the average return and standard deviation of REIT X using the data provided.

Example 1 (Answer)

Year	Return (r)	$r - \bar{r}$	$(r - \bar{r})^2$
1	10	2	4
2	30	22	484
3	-20	-28	784
4	0	-8	64
5	20	12	144
Average ($\bar{r}$) =	8	Sum =	1480
	Sample Variance (σ^2) =		370
	Sample Standard Deviation (σ) =		19.24
	Population Variance (σ^2) =		296
	Population Standard Deviation (σ) =		17.20

Example 2

Year	Return (%)
2004	15
2005	20
2006	25
2007	-10
2008	12
2009	18
2010	22

Calculate the following performance measures for Property Fund X using the data provided:

- a) Average return
- b) Variance
- c) Standard deviation

Example 2 (Answer)

Year	Return (r)	$r - \bar{r}$	$(r - \bar{r})^2$
2004	15	0.43	0.18
2005	20	5.43	29.47
2006	25	10.43	108.76
2007	-10	-24.57	603.76
2008	12	-2.57	6.61
2009	18	3.43	11.76
2010	22	7.43	55.18
Average =	14.57	Sum =	815.71
	Sample Variance (σ^2) =		135.95
	Sample Standard Deviation (σ) =		11.66
	Population Variance (σ^2) =		116.53
	Population Standard Deviation (σ) =		10.79

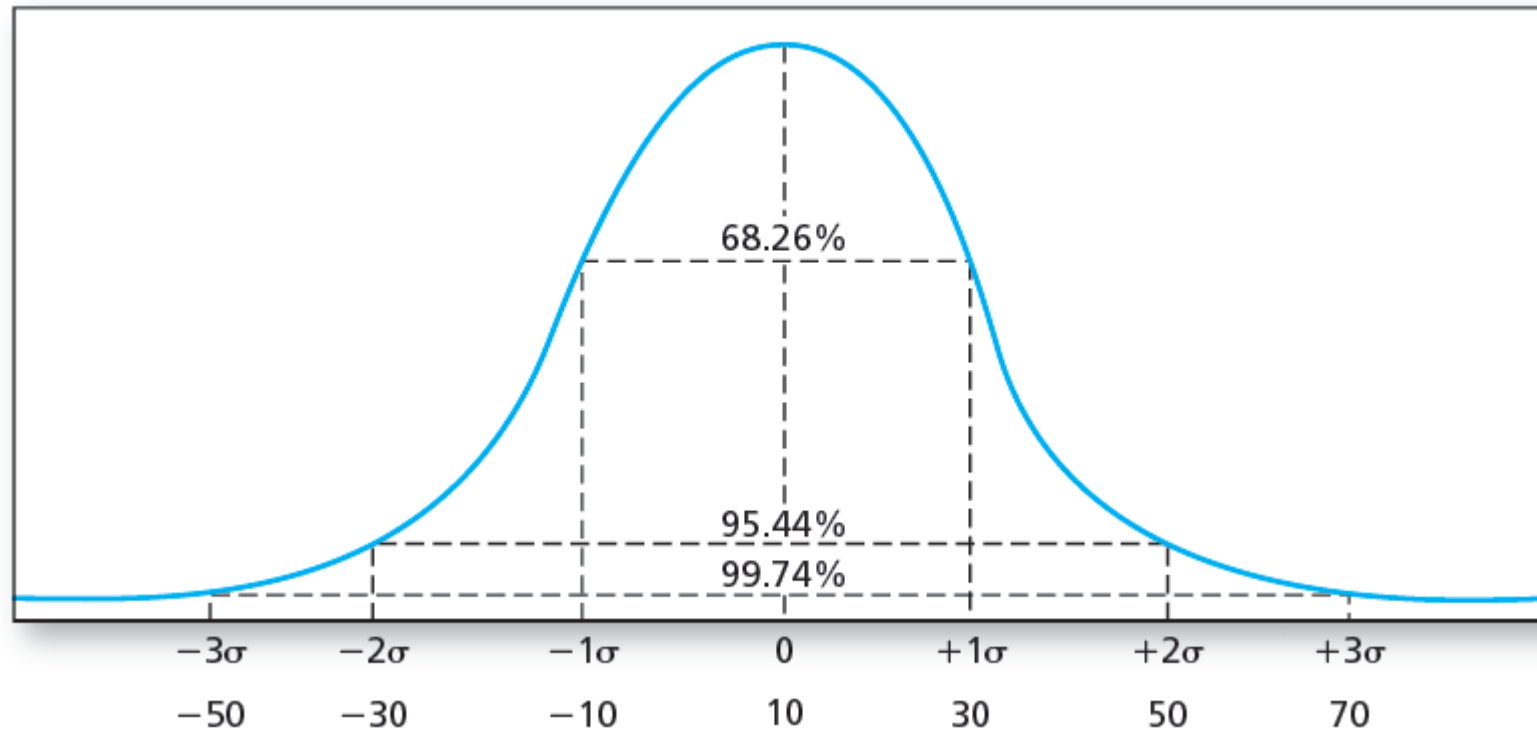
Using the Standard Deviation as a Measure of Risk

Standard deviation is a good measure of risk when returns are symmetric

Future scenarios can be estimated using only the mean and the standard deviation

The dependence of returns across securities can also be summarized using only the pairwise correlation coefficients

The Normal Distribution



Mean return= 10%, SD = 20%

The Normal Distribution

If asset returns are normally distributed, we can convert them into standard normal distributions using the following formula:

$$\text{Z - score} = \frac{r - \bar{r}}{\sigma}$$

Where:

r = observed return

$\bar{r}$ = mean return

σ = standard deviation

The Normal Distribution

Once we have calculated the z-score, we can look up the probability that corresponds to this z-score from a z-table (normal distribution table).

The first two digits of the z-value are in the first column of the z-table.

The third digit of the z-value is in the first row of the z-table.

For example, for a z-value of 1.13, the first two digits, 1.1 can be found on the first column. Once you locate 1.1 in the first column, you move horizontally to the right to column with heading 0.03, the 3 being the third digit of the z-value. The probability value here is 0.8708.

In Excel, you can use the following function to obtain the probability value:

`=NORM.S.DIST(1.13,TRUE) = 0.8708`

Example 1

Year	Return
1	10%
2	30%
3	-20%
4	0%
5	20%

- Calculate the average return and sample and population standard deviation of REIT X using the data provided.
- Describe the implication of the results in (a) if we assume that the returns of REIT X are normally distributed.

Example 1a (Answer)

Year	Return (r)	$r - \bar{r}$	$(r - \bar{r})^2$
1	10	2	4
2	30	22	484
3	-20	-28	784
4	0	-8	64
5	20	12	144
Average ($\bar{r}$) =		8	Sum = 1480
Sample Variance (σ^2) =			370
Sample Standard Deviation (σ) =			19.24
Population Variance (σ^2) =			296
Population Standard Deviation (σ) =			17.20

Example 1b (Answer)

The following probabilities give the range within which returns will fall;

Probability	Range	$R_A=8\%; s=19.2\%$
68%	$R_A \pm 1\sigma$	-11.2 to +27.2
95%	$R_A \pm 2\sigma$	- 30.4 to +46.4
99%	$R_A \pm 3\sigma$	- 49.6 to +65.6

Example 2

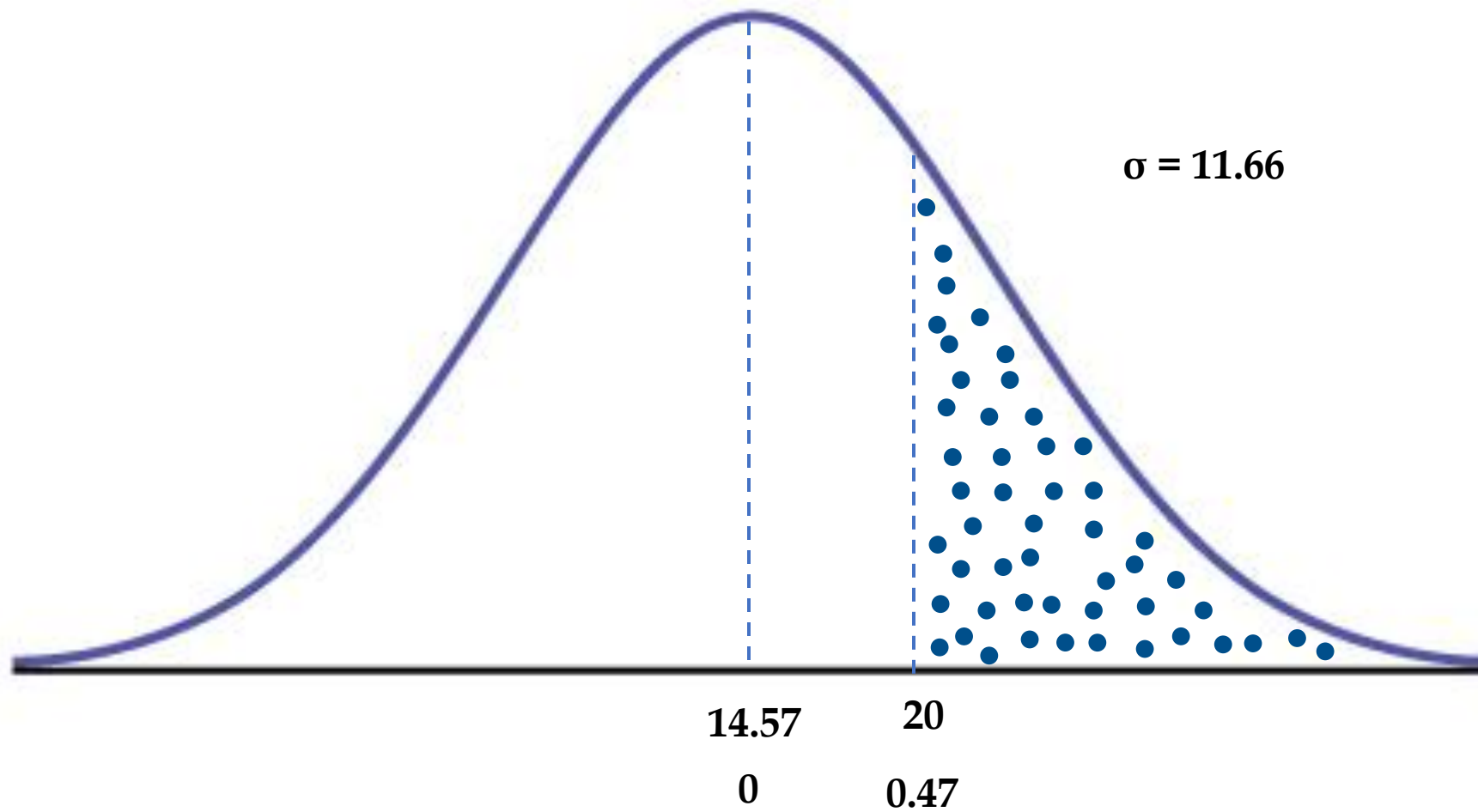
Year	Return (%)
2004	15
2005	20
2006	25
2007	-10
2008	12
2009	18
2010	22

- a) Calculate the average return and sample and population standard deviation of Property Fund X using the data provided.
- b) Assuming property returns are normally distributed, what is the probability that the returns for a particular year will be greater than the benchmark return of 20%? (Use the sample standard deviation for this calculation)

Example 2a (Answer)

Year	Return (r)	$r - \bar{r}$	$(r - \bar{r})^2$
2004	15	0.43	0.18
2005	20	5.43	29.47
2006	25	10.43	108.76
2007	-10	-24.57	603.76
2008	12	-2.57	6.61
2009	18	3.43	11.76
2010	22	7.43	55.18
Average	14.57	Sum	815.71
	Sample Variance (σ^2)		135.95
	Sample Standard Deviation (σ)		11.66
	Population Variance (σ^2)		116.5306
	Population Standard Deviation (σ)		10.7949

Example 2b (Answer)



Example 2b (Answer)

Average ($\bar{r}$) = 14.57%

Standard Deviation (σ) = 11.66%

$r = 20\%$

$$\text{Z - score} = \frac{r - \bar{r}}{\sigma} = \frac{20 - 14.57}{11.66} = 0.47$$

A z-score of 0.47 corresponds to a probability of 68.08

Since this is the probability of obtaining returns up to 20%, the probability of obtaining returns greater than 20% would be:

$$\text{Probability of return greater than } 20\% = 1 - 0.6808 = 0.3192$$

Example 3

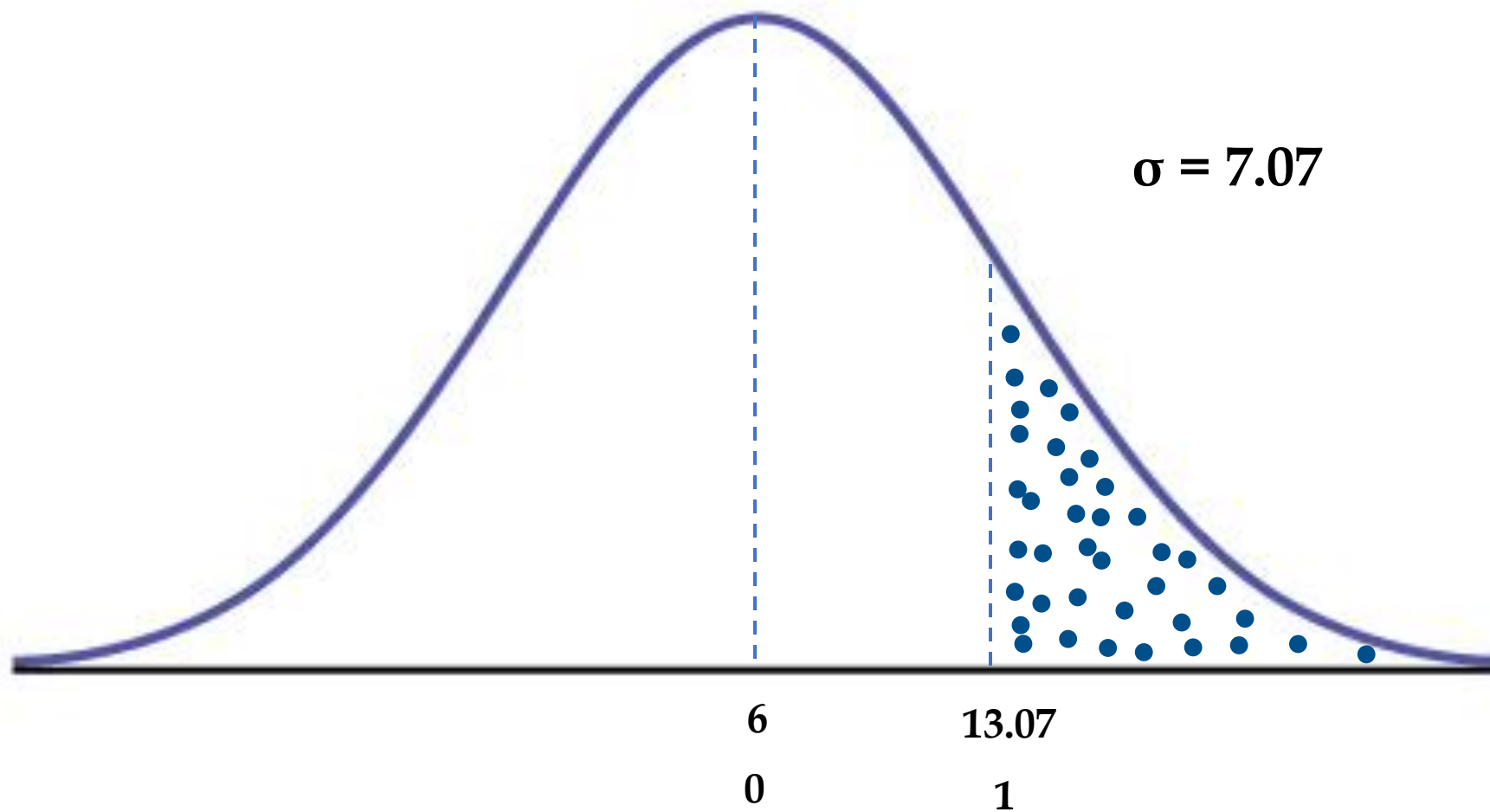
XYZ Property Fund had returns of 4%, 7%, -2% and 15% over the last four years.

- a) Calculate the sample standard deviation of the returns given.
- b) What is the probability that the next return would be greater than the benchmark return of 13.07%?
- c) What is the probability that the fund would not produce a negative return in any year?

Example 3a (Answer)

Year	Return (r)	$r - \bar{r}$	$(r - \bar{r})^2$
1	4	-2	4
2	7	1	1
3	-2	-8	64
4	15	9	81
Average	6.00	Sum	150
	Sample Variance (σ^2)		50
	Sample Standard Deviation (σ)		7.07

Example 3b (Answer)



Example 3b (Answer)

Average ($\bar{r}$) = 6%

Standard Deviation (σ) = 7.07%

$r = 13.07\%$

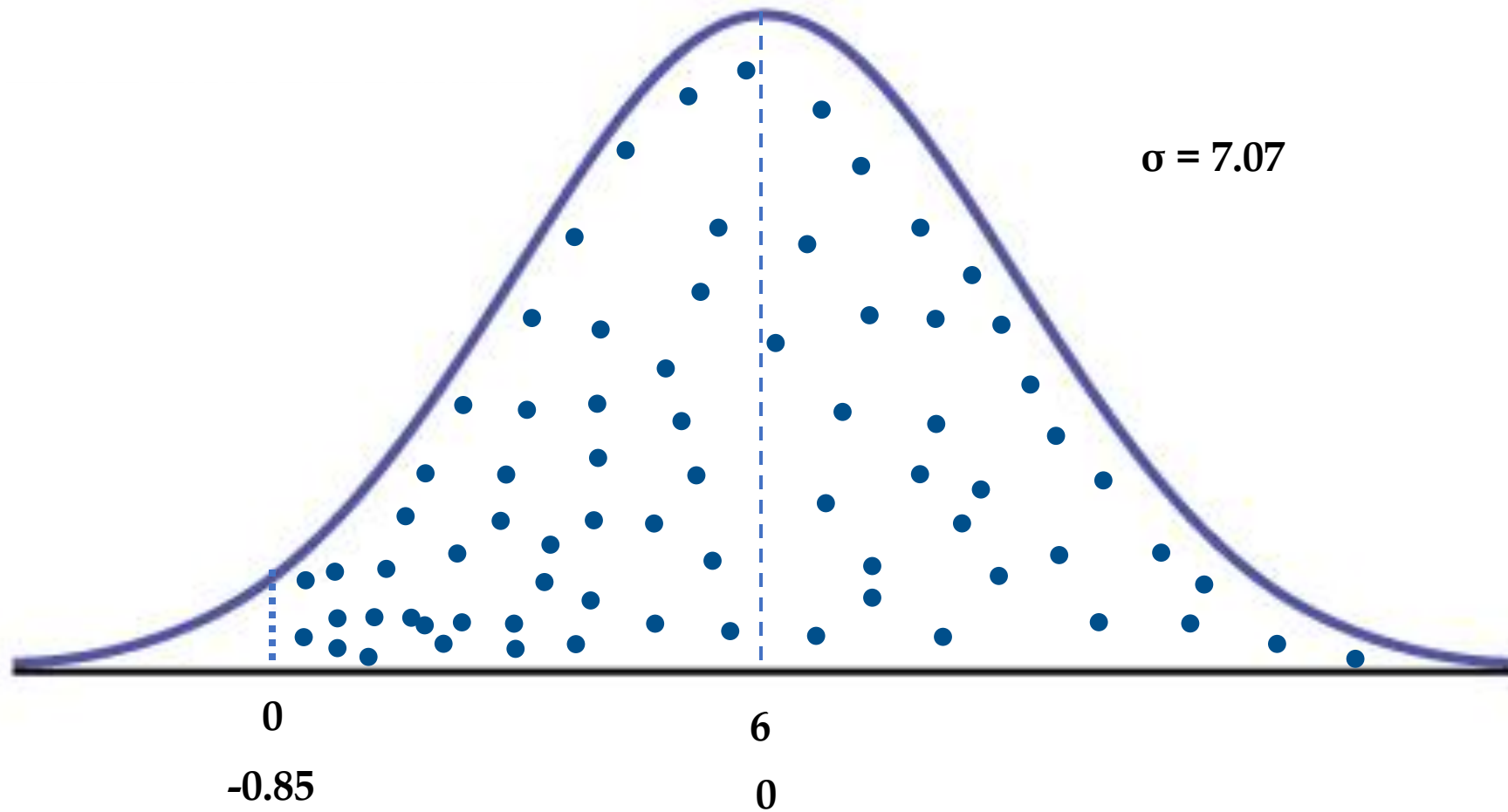
$$\text{Z - score} = \frac{r - \bar{r}}{\sigma} = \frac{13.07 - 6}{7.07} = 1$$

A z-score of 1 corresponds to a probability of 0.8413

Since this is the probability of obtaining returns up to 13.07%, the probability of obtaining returns greater than 13.07 would be:

Probability of return greater than 13.07 = $1 - 0.8413 = 0.1587$

Example 3c (Answer)



Example 3c (Answer)

Average ($\bar{r}$) = 6%

Standard Deviation (σ) = 7.07%

$r = 0\%$

$$\text{Z - score} = \frac{r - \bar{r}}{\sigma} = \frac{0 - 6}{7.07} = -0.85$$

A z-score of -0.85 corresponds to a probability of 0.1977

Since this is the probability of obtaining returns up to 0%, the probability of obtaining returns greater than 0% would be:

$$\text{Probability} = 1 - 0.1977 = 0.8023 = 80.23\%$$

Example 4

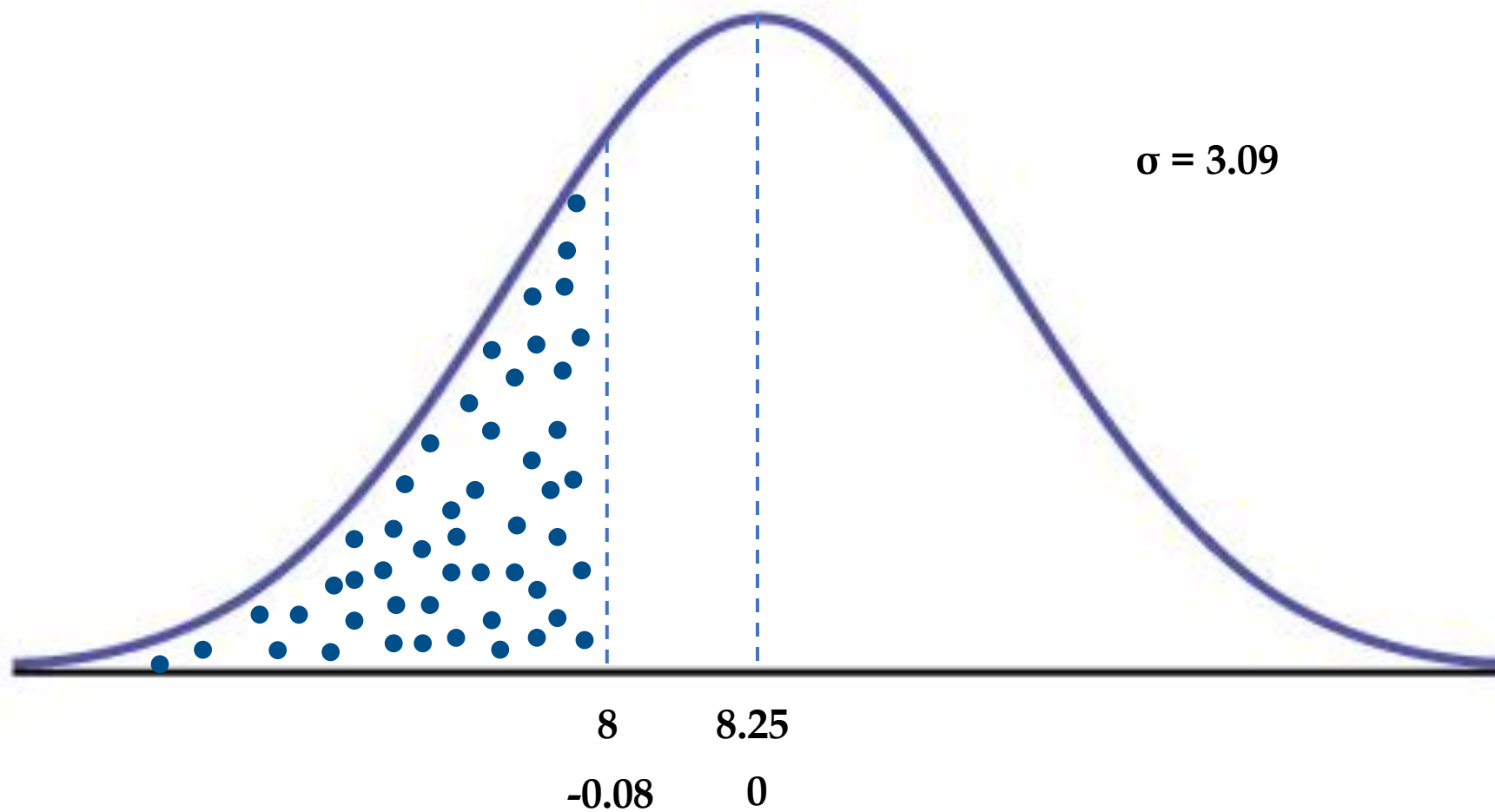
Year	Return (r)
2009	7
2010	11
2011	4
2012	6
2013	9.5
2014	12

- a) Calculate the average return and standard deviation of Propco Investment Fund using the data provided.
- b) What is the probability that the returns of Propco would fall below the benchmark return of 8%?
- c) What is the probability that Propco's return will fall between 8% and 10%?

Example 4a (Answer)

Year	Return (r)	$r - \bar{r}$	$(r - \bar{r})^2$
2009	7	-1.25	1.56
2010	11	2.75	7.56
2011	4	-4.25	18.06
2012	6	-2.25	5.06
2013	9.5	1.25	1.56
2014	12	3.75	14.06
Average ($\bar{r}$) =	8.25	Sum =	47.88
	Sample Variance (σ^2) =		9.58
	Sample Standard Deviation (σ) =		3.09

Example 4b (Answer)



Example 4b (Answer)

Average ($\bar{r}$) = 8.25%

Standard Deviation (σ) = 3.09%

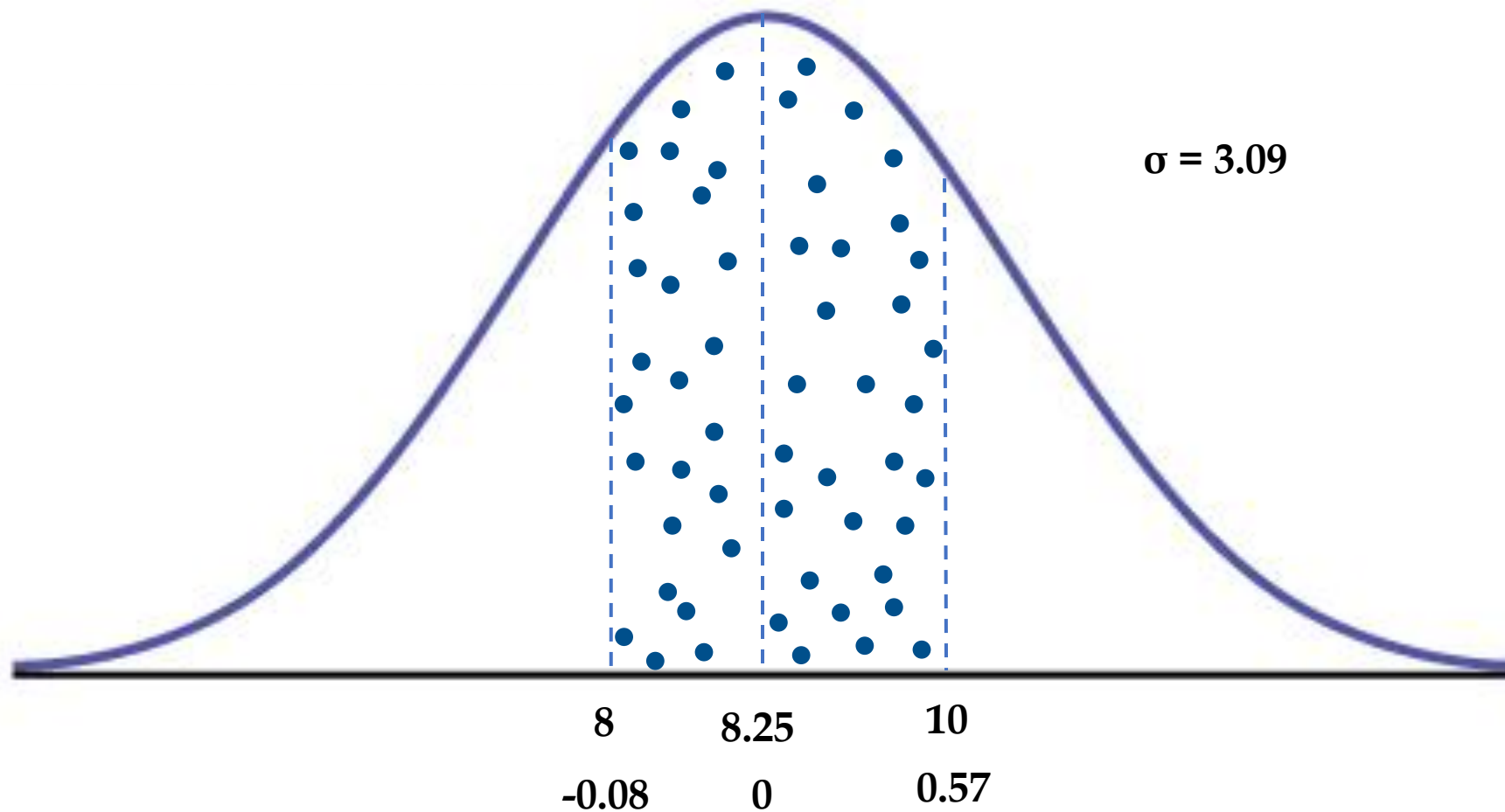
$r = 8\%$

$$\text{Z - score} = \frac{r - \bar{r}}{\sigma} = \frac{8 - 8.25}{3.09} = -0.0808$$

A z-score of -0.0808 corresponds to a probability of 0.4678

Therefore, the probability that returns will fall below 8% is 46.78%

Example 4c (Answer)



Example 4c (Answer)

Average ($\bar{r}$) = 8.25

Standard Deviation (σ) = 3.09

$r = 8\%$

$$\text{Z - score} = \frac{r - \bar{r}}{\sigma} = \frac{8 - 8.25}{3.09} = -0.0808$$

A z-score of -0.0808 corresponds to a probability of 0.4678

Average ($\bar{r}$) = 8.25

Standard Deviation (σ) = 3.09

$r = 10\%$

$$\text{Z - score} = \frac{r - \bar{r}}{\sigma} = \frac{10 - 8.25}{3.09} = 0.5655$$

A z-score of 0.5655 corresponds to a probability of 0.7141

Probability that returns will fall between 8 and 10% is:

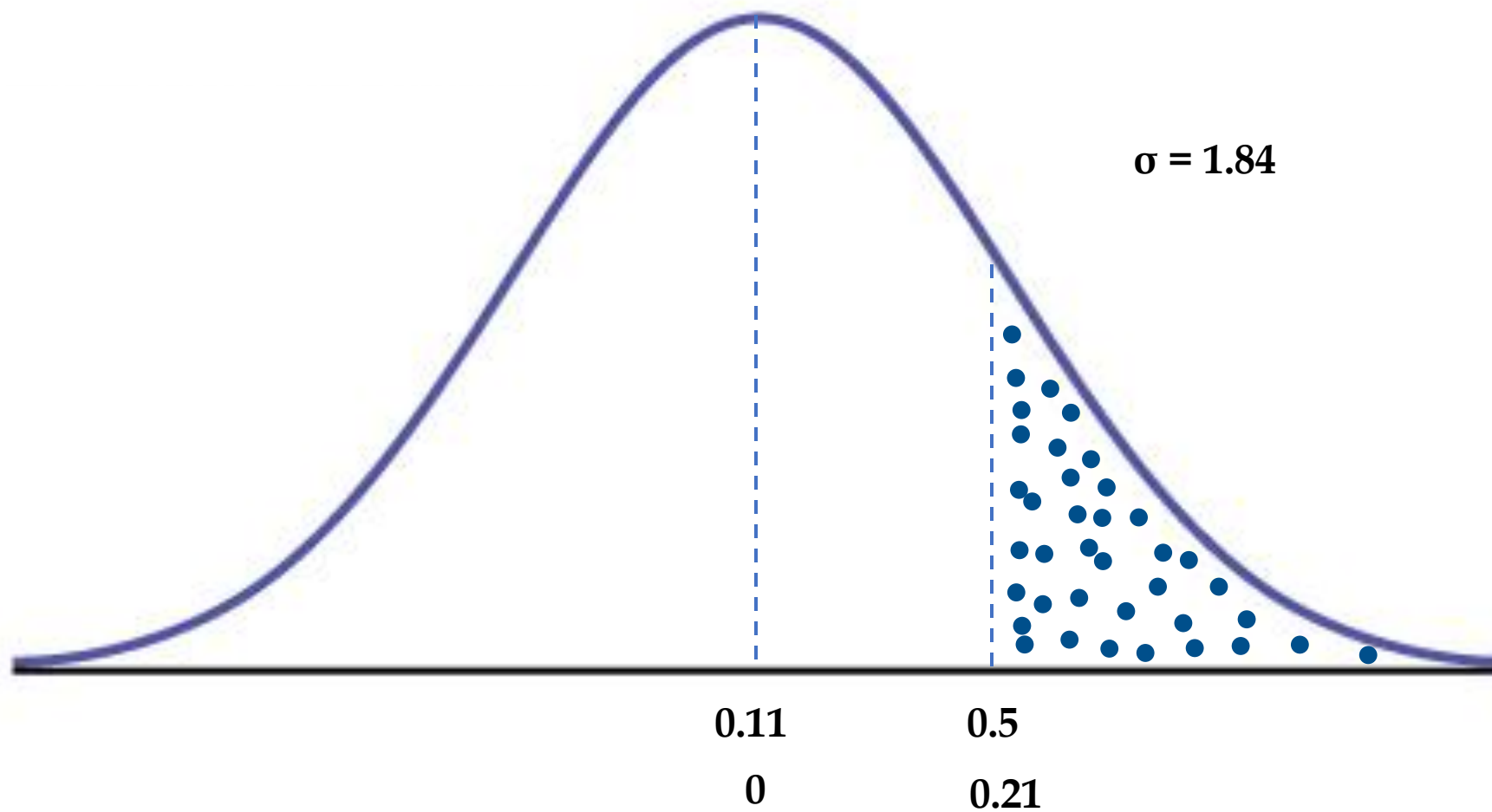
$$0.7141 - 0.4678 = 0.2463 = \mathbf{24.63\%}$$

Example 5

The daily returns of Axis Property Fund, recorded over the past few days, had an average return of 0.11% and a standard deviation of 1.84%. Using this information, and assuming that the returns of Axis Property Fund are normally distributed, answer the following questions:

- a) What is the probability, for any given day, of a return greater than 0.5%
- b) What is the probability for any given day, of a loss greater than 2%
- c) What is the probability for any given day, of a return between 0% and 1%
- d) What is the probability, for any given day, of a positive return or loss greater than 3%?

Example 5a (Answer)



Example 5a (Answer)

What is the probability, for any given day, of a return greater than 0.5%:

Average ($\bar{r}$) = 0.11%

Standard Deviation (σ) = 1.84%

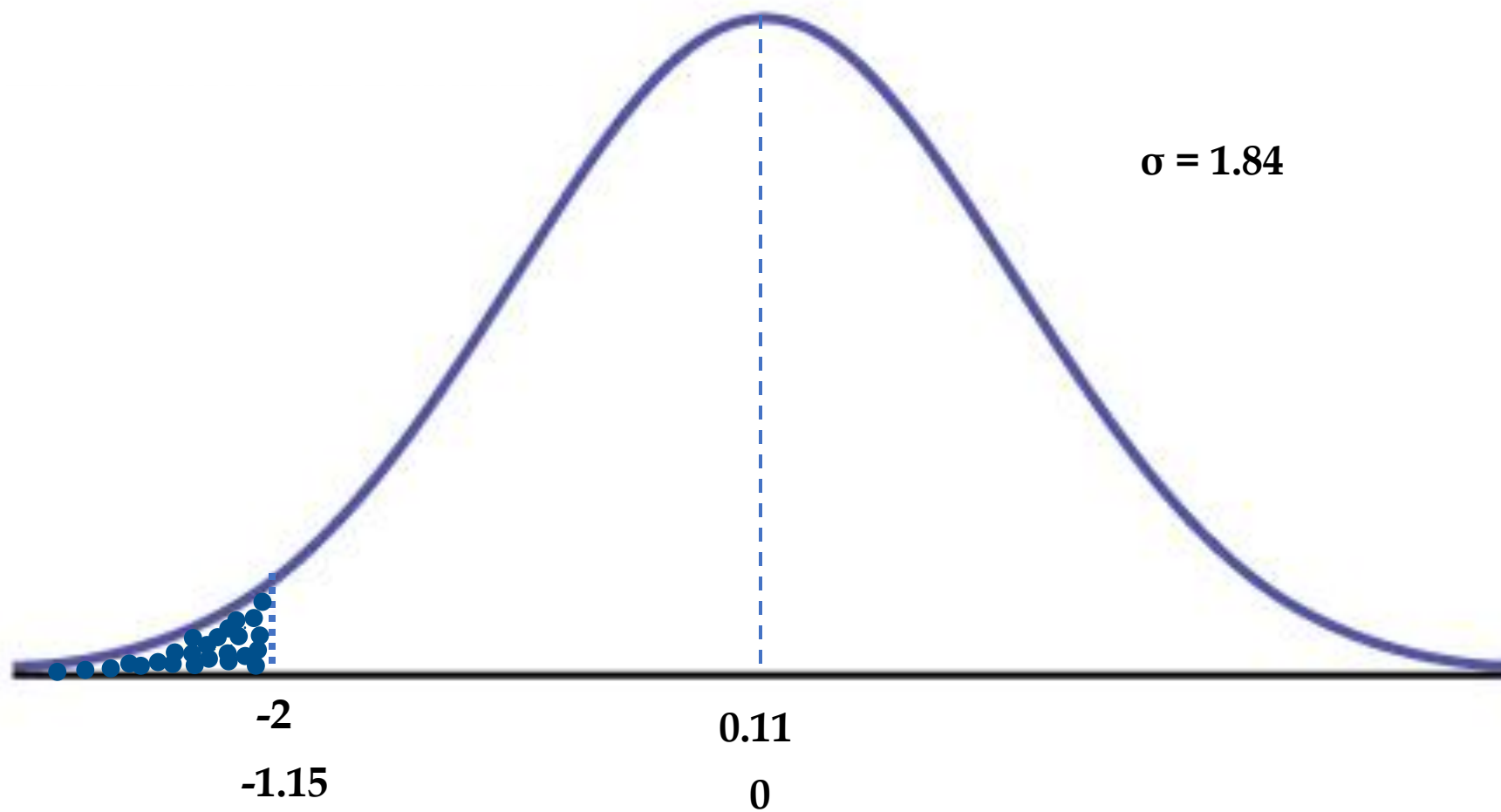
$r = 0.5$

$$\text{Z - score} = \frac{r - \bar{r}}{\sigma} = \frac{0.5 - 0.11}{1.84} = 0.21$$

A z-score of 0.21 corresponds to a cumulative probability of 0.5832

Therefore, the probability that returns will be greater than 0.5% = $1 - 0.5832 = 0.4168$.

Example 5b (Answer)



Example 5b (Answer)

What is the probability for any given day, of a loss greater than 2%:

Average ($\bar{r}$) = 0.11%

Standard Deviation (σ) = 1.84%

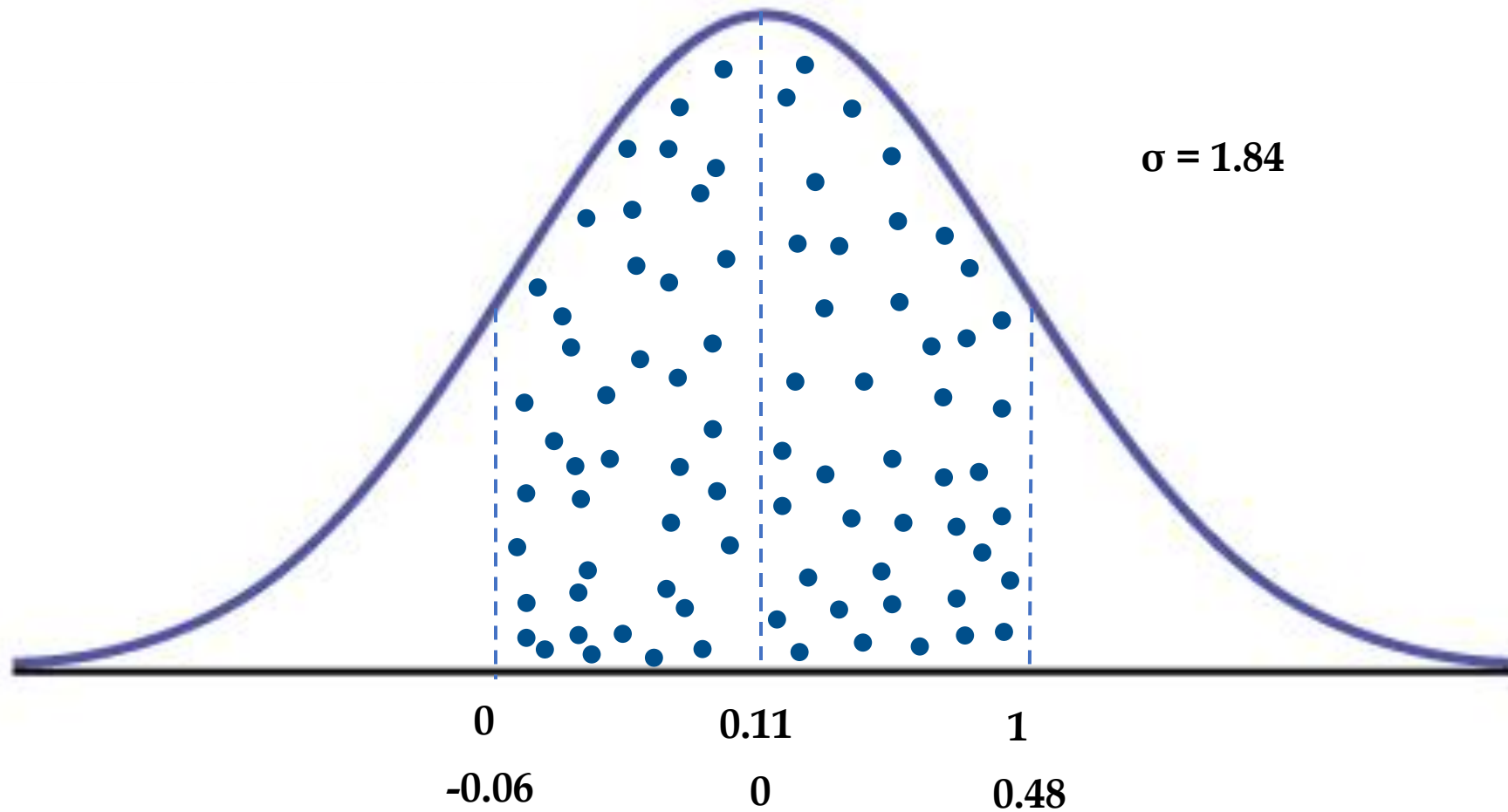
$r = -2\%$

$$\text{Z - score} = \frac{r - \bar{r}}{\sigma} = \frac{-2 - 0.11}{1.84} = -1.15$$

A z-score of -1.15 corresponds to a cumulative probability of 0.1251

Therefore, the probability that returns will be lower than -2% = **0.1251**

Example 5c (Answer)



Example 5c (Answer)

Probability of returns being less than 0%:

Average ($\bar{r}$) = 0.11%

Standard Deviation (σ) = 1.84%

$r = 0\%$

$$\text{Z - score} = \frac{r - \bar{r}}{\sigma} = \frac{0 - 0.11}{1.84} = -0.06$$

A z-score of -0.06 corresponds to a cumulative probability of 0.4761

Probability of returns being less than 1%:

Average ($\bar{r}$) = 0.11%

Standard Deviation (σ) = 1.84%

$r = 1\%$

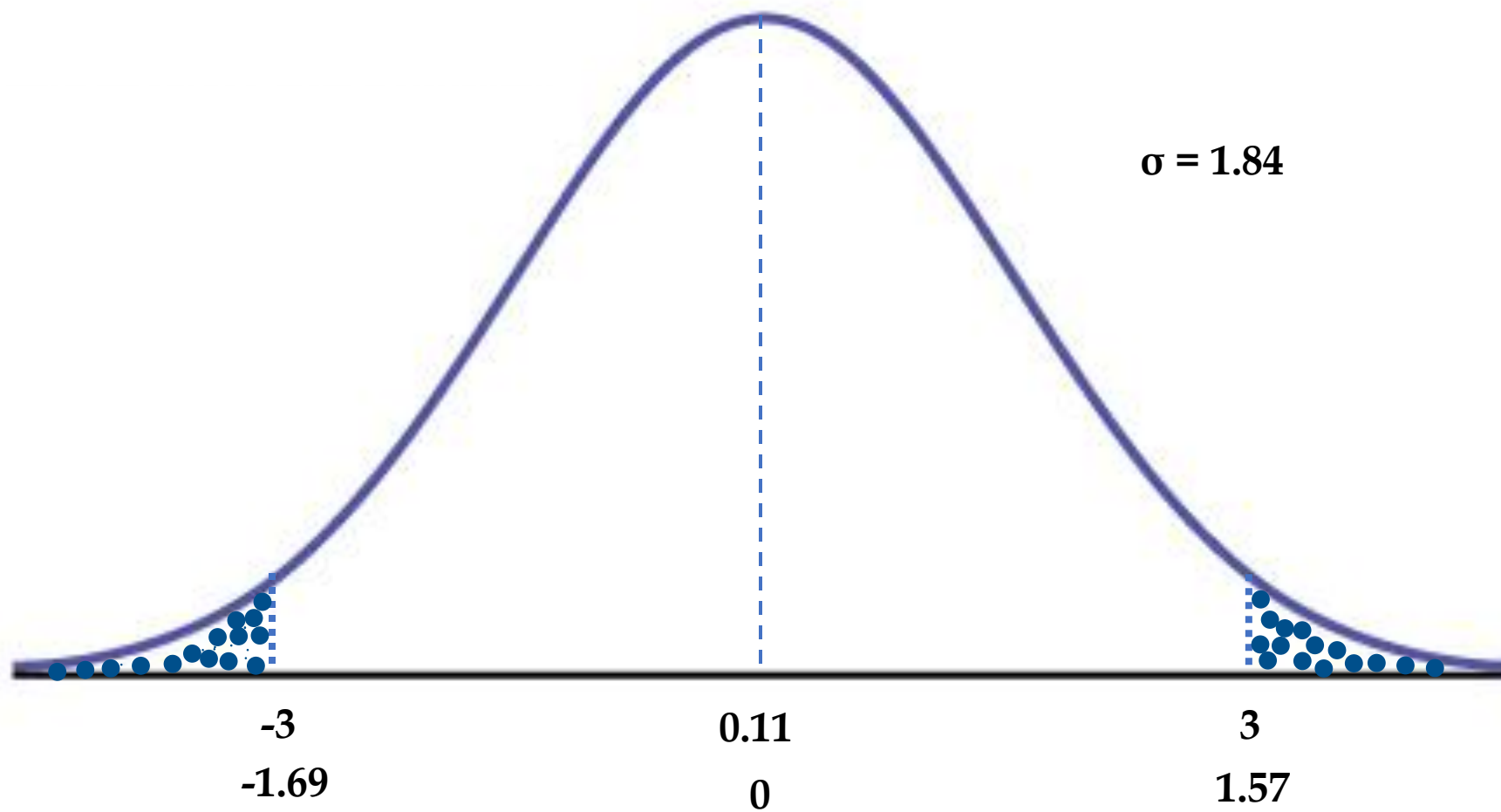
$$\text{Z - score} = \frac{r - \bar{r}}{\sigma} = \frac{1 - 0.11}{1.84} = 0.48$$

A z-score of 0.48 corresponds to a cumulative probability of 0.6844

Probability of returns falling between 0 and 1% would be:

$$0.6844 - 0.4761 = 0.2083$$

Example 5d (Answer)



Example 5d (Answer)

Probability of returns being less than -3%:

Average ($\bar{r}$) = 0.11%

Standard Deviation (σ) = 1.84%

$r = -3\%$

$$\text{Z - score} = \frac{r - \bar{r}}{\sigma} = \frac{-3 - 0.11}{1.84} = -1.69$$

A z-score of -1.69 corresponds to a cumulative probability of 0.0455

Probability of returns being greater than +3%:

Average ($\bar{r}$) = 0.11%

Standard Deviation (σ) = 1.84%

$r = 3\%$

$$\text{Z - score} = \frac{r - \bar{r}}{\sigma} = \frac{3 - 0.11}{1.84} = 1.57$$

A z-score of 1.57 corresponds to a cumulative probability of 0.9406

Probability of returns falling above 3% would be $1 - 0.9406 = 0.0594$

Probability of returns being less than -3% or greater than +3% :
 $0.0455 + 0.0581 = 0.1036$